



Solving Rational Inequalities

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A rational inequality is an inequality which contains a rational expression. When solving these rational inequalities, there are steps that lead us to the solution.

To solve Rational Inequalities:

- (1) Write the inequality as an $\frac{ax+b}{cx+d} < 0$, and solve the $\frac{ax+b}{cx+d} = 0$.
- (2) Determine any values that make the denominator equal 0.
- (3) On a number line, mark each of the critical values from steps 1 and 2. These values will create intervals on the number line.
- (4) Select a test point in each interval, and check to see if that test point satisfies the inequality. (Find the intervals which satisfy the inequality).
- (5) Mark the number line to reflect the values and intervals that satisfy the inequality.
- (6) State your answer using the desired form of notation.

*What does "critical value" mean?

Example 1: Solve $\frac{x-4}{x+5} < 4$

$$\frac{x-4}{x+5} = 4$$

$$\frac{x-4}{1} = 4$$

*What does "satisfy" mean in math?

Create an equation. Change $<$ to $=$, and solve. Notice that if $x = -5$, the denominator is 0.

Multiply both sides by $x+5$ to eliminate the fraction.

In a proportion the product of the means equals the product of the extremes.

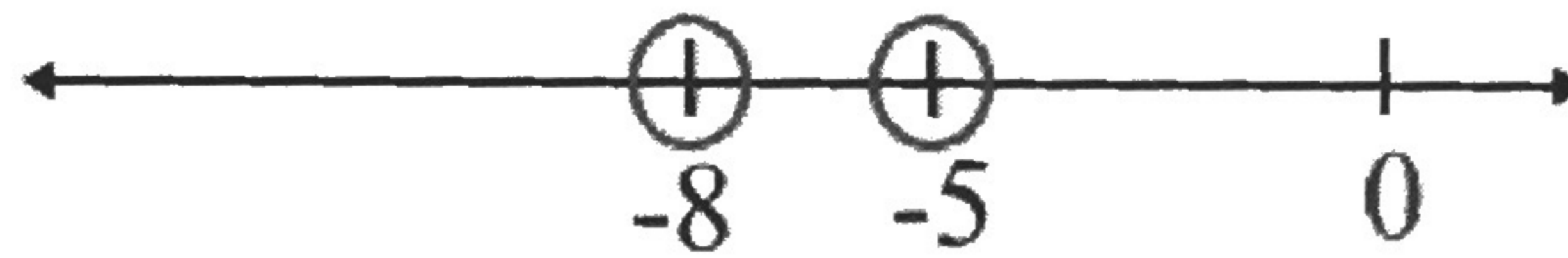
Critical values are

$$x = -8 \text{ and}$$

$$x = -5 \leftarrow \text{why is negative 5 a critical value?}$$

On the number line, plot -5 and -8. Since -5 cannot be used, it is an open circle.

The inequality is strictly "less than", so the -8 is also an open circle.



Test a point in each of the three intervals formed: **place the points -9, -6, and 0 on your numberline.*

test point -9	test point -6	test point 0
$\frac{-4}{+5} < 4$	$\frac{-4}{+5} < 4$	$\frac{-4}{+5} < 4$

Stated as an inequality, the solution is:

x or x

Stated in interval notation, the solution is:

$() \cup ()$

When the numerator of the inequality is a \dots , combine the Quadratic Inequality method of solution with this Rational Inequality method. Check out this example.

Example 2: Solve $\frac{x^2 - 2x - 15}{x - 2} \geq 0$

$$\frac{x^2 - 2x - 15}{x - 2} \geq 0$$

\geq the numerator, form an equation, and solve this quadratic equation.

Factor, and find the solutions or critical values for the numerator.

Also keep in mind that the

$$x^2 - 2x - 15 = 0$$

$$(\quad)(\quad) = 0$$

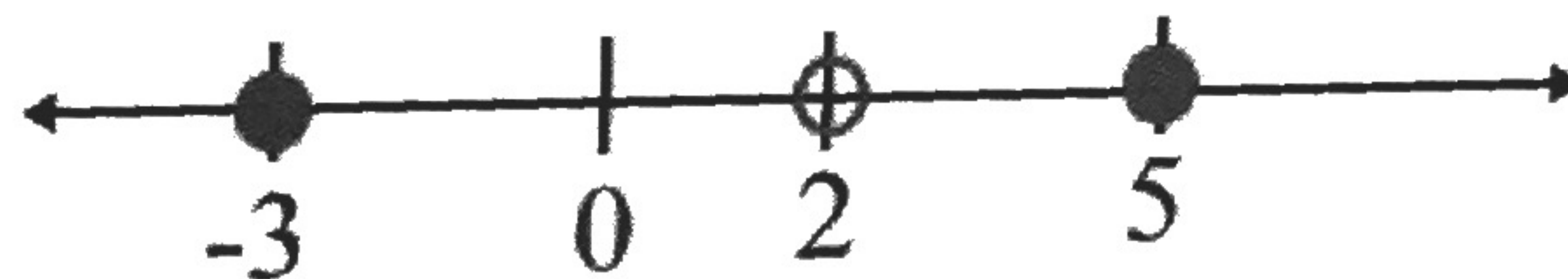
OR

denominator has $x = 2$ *Why is 2 as a value as well. An important value

Since $x = 2$ creates an undefined, it is drawn as an open circle on the number line.

Place the critical values on a number line. Since the inequality is *greater than or equal to*, the $x =$ or $x =$ are drawn on the number line as a closed circle, which means to include them as part of the answer. Test the intervals that are formed.

*Why is 3 and 5 colored in?



*Why is 2 an open circle?

test $x = -4$	test $x = 0$	test $x = 3$	test $x = 6$
$\frac{(\quad)^2 - 3(\quad) - 15}{(\quad) - 2} \geq 0$	$\frac{(\quad)^2 - 3(\quad) - 15}{(\quad) - 2} \geq 0$	$\frac{(\quad)^2 - 3(\quad) - 15}{(\quad) - 2} \geq 0$	$\frac{(\quad)^2 - 3(\quad) - 15}{(\quad) - 2} \geq 0$

The solution is:

$$x \leq -3 \text{ or } x \geq 5$$

In interval notation, the solution is:

$$[-3, \infty) \cup (2, 5, \infty)$$